THE ASYMPTOTIC BEHAVIOR OF THE $\pi^0\gamma^\star\gamma$ TRANSITION

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It is discussed how various *Ansätze* for the dressed quark-photon ($qq\gamma$) vertices $\Gamma^\mu(q,q')$ influence the asymptotics of the $\gamma^\star\gamma \to \pi^0$ transition form factor.

It has been shown [1] that the Schwinger-Dyson (SD) approach to physics of quarks and gluons leads to the $\gamma^\star(k)\gamma(k') \to \pi^0(p)$ transition form factor $T_{\pi^0}(k^2, {k'}^2)$ which has the asymptotic momentum dependence

$$T_{\pi^0}(-Q^2, 0) = \frac{\mathcal{K}}{Q^2} \qquad (\mathcal{K} \to \text{const as } Q^2 \to \infty), \tag{1}$$

for large spacelike $k^2 = -Q^2 < 0$. This is consistent with the data [2] at the presently largest accessible Q^2 and in agreement (up to the precise value of $\mathcal{K}$) with perturbative QCD where [3] $\mathcal{K} = 2f_\pi$, operator product expansion (OPE) where [4] $\mathcal{K} = 4f_\pi/3$, and QCD sum rules where [5] $\mathcal{K} \approx 1.6f_\pi$.

The SD approach, where the quark propagators $S(q) = [A(q^2)\not{q} - B(q^2)]^{-1}$ are dynamically dressed, also requires consistently dressed quark-photon ($qq\gamma$) vertices $\Gamma^\mu(q,q')$ in order to satisfy the vector Ward-Takahashi identity (WTI). [In addition to using for the pseudoscalar vertex the quark-antiquark pion Bethe-Salpeter (BS) bound-state vertex $\Gamma_{\pi^0}(q,p)$, this defines *generalized impulse approximation* (GIA).] Even just approximately adequate SD solutions for $\Gamma^\mu(q,q')$ are not yet available, and in practice the more or less realistic WTI-satisfying *Ansätze* still must be used.

The topic of this writeup is the dependence of the asymptotic coefficient $\mathcal{K}$ on the choice of the *Ansatz* for $\Gamma^\mu(q,q')$. This topic needs clarification, since Ref. [6] expressed a slight disagreement about one detail in what we found [1]. (The rest of the material of my talk is amply covered in Refs. [1,7,8,9].)

Ref. [1] showed that the SD approach predicts $\mathcal{K} = 4f_\pi/3$ (the same as OPE [4]) for *all* $qq\gamma$ vertices $\Gamma^\mu(q',q)$ which go into the bare one (γ^μ) even if just *one* of the squared momenta q^2 or ${q'}^2$ becomes infinite. This was illustrated on the examples of the Curtis-Pennington (CP) vertex [10] Γ^μ_{CP} and the modified Curtis-Pennington (mCP) vertex Γ^μ_{mCP} [1]. (For the latter, $T_{\pi^0}(-Q^2,0)$ was calculated also for finite values of Q^2.) Both the CP and mCP vertices are

multiplicatively renormalizable, so that our result [1] on the asymptotic behavior of $T_{\pi^0}(-Q^2, 0)$ subsequently received further support from Ref. [11]. This reference generalized our derivation [1] by taking into account renormalization explicitly, showing that the asymptotics of Ref. [1] with $\mathcal{K} = 4f_\pi/3$ must hold for any $qq\gamma$ vertex which is consistent with multiplicative renormalizability.

However, Ref. [1] also showed that the usage of the "minimal" WTI-satisfying $qq\gamma$ vertex Γ^μ_{BC}, namely the Ball-Chiu (BC) one, leads to the asymptotic coefficient $\mathcal{K} = 4\widetilde{f}_\pi/3$, where $\widetilde{f}_\pi$ is the quantity given by the same Mandelstam-formalism expression as the pion decay constant f_π, except that the integrand is modified by the factor $[1 + A(q^2)]^2/4$. (In the case of our solutions [7], this gives $\widetilde{f}_\pi = 1.334 f_\pi = 124$ MeV.) Note that the arguments of Ref. [11] do not preclude the change $f_\pi \to \widetilde{f}_\pi$, since the BC vertex is *not* consistent with multiplicative renormalizability [10]. This modification of $\mathcal{K}$ is caused by the different asymptotic behavior of the BC vertex, which tends to the bare vertex, $\Gamma^\mu_{BC}(q', q) \to \gamma^\mu$, only when the squared momenta in *both* fermion legs tend to infinity, $q'^2, q^2 \to \pm\infty$. The origin of the factor $(1/2)^2[1 + A([q + p/2]^2)][1 + A([q - p/2]^2)] \approx [1 + A(q^2)]^2/4$ modifying the integrand when $\Gamma^\mu(q', q) = \Gamma^\mu_{BC}(q', q)$ is then clear: $T_{\pi^0}(k^2, k'^2)$ is extracted from the tensor amplitude $T^{\mu\nu}_{\pi^0}(k, k')$ for the GIA triangle diagram,

$$T^{\mu\nu}_{\pi^0}(k,k') \propto \int \frac{d^4q}{(2\pi)^4} \mathrm{tr}\{\Gamma^\mu(q - \frac{p}{2}, k + q - \frac{p}{2}) S(k + q - \frac{p}{2})$$

$$\times \Gamma^\nu(k + q - \frac{p}{2}, q + \frac{p}{2}) S(q + p/2) \Gamma_{\pi^0}(q, p) S(q - p/2)\} + (k \leftrightarrow k', \mu \leftrightarrow \nu)\,, \quad (2)$$

and since all quark loop momenta q contribute, the small values of $(q \pm p/2)^2 \approx q^2$ in one quark leg will prevent the BC vertex $\Gamma^\mu_{BC}(q', q)$ from reducing always to the bare γ^μ-vertex, even when a hard virtual photon momentum $k^2 = -Q^2$ makes "bare" the other fermion leg in the vertices $\Gamma^\mu(q', q) = \Gamma^\mu_{BC}(q', q)$.

However, this result on the asymptotics of $T_{\pi^0}(-Q^2, 0)$ when using the BC vertex, caused some controversy since Ref. [6] claimed that even for the BC vertex $\mathcal{K} = 4f_\pi/3$, *i.e.*, that no modification occurs for the BC vertex due to one soft quark leg. The argument of Ref. [6] (see its Sec. *4.*) is that there are in fact no soft legs in the $qq\gamma$ vertices when Q^2 becomes very large. The on-shell condition for the pion and one photon, $(p - k)^2 = M_\pi^2 - 2p \cdot k - Q^2 \approx -2p \cdot k - Q^2 = k'^2 = 0$, and $k^2 = -Q^2$, are used to argue that the pion momentum p has components which must scale like k and thus like Q. Then, $A([q \pm p/2]^2) = A(q^2 \pm q \cdot p + M_\pi^2)$ would tend to 1 as $Q^2 \to \infty$ even for very soft loop momenta q, just because of $p \sim Q$, causing $\Gamma^\mu_{BC} \to \gamma^\mu$.

We will now demonstrate that this argument does not hold. The very fact that the size of the *components* is invoked makes the argument suspect,

because it is a frame-dependent statement. The argument of Ref. [6] relies on working in a Lorentz frame such as the one where $k = (0, 0, 0, \sqrt{Q^2})$, $k' = (E_\pi, 0, 0, -E_\pi)$, $p = (E_\pi, 0, 0, \sqrt{Q^2} - E_\pi)$, and $E_\pi = (Q^2 + M_\pi^2)/2\sqrt{Q^2}$. Even if one sticks just to that choice in one's calculation, one can expect persistent soft contributions because of those soft loop momenta q which are also perpendicular to p so that $p \cdot q = 0$. However, the shortest and clearest demonstration that, at least in this application, $q \cdot p$ cannot be hard if q is soft, is noting that one can make a Lorentz transformation to the pion rest frame. In this case it is the boost transformation along the z-axis and with the parameter $\beta = (Q^2 - M_\pi^2)/(Q^2 + M_\pi^2)$. In that frame, $k = (M_\pi - E_\gamma, 0, 0, E_\gamma)$ and $k' = (E_\gamma, 0, 0, -E_\gamma)$, with $E_\gamma = (Q^2 + M_\pi^2)/2M_\pi$, whereas $p = (M_\pi, 0, 0, 0)$, making it clear that for the light pion, $A([q \pm p/2]^2)$ is approximated well by $A(q^2)$ and not by $A(\pm q \cdot p)$ which allegedly [6] would be 1.

We want to make clear that we of course give precedence to the value $\mathcal{K} = 4f_\pi/3$ for the asymptotic coefficient as the one having the more fundamental meaning, resulting from the $qq\gamma$ vertices such as the CP or mCP ones, which have properties closer to the true vertex solution, such as being renormalizable. Also indicative is the asymptotics found by Ref. [1] for the case when both photons are off-shell, $k^2 = -Q^2 << 0$ and $k'^2 = -Q'^2 \leq 0$:

$$T_{\pi^0}(-Q^2, -Q'^2) = \frac{4}{3} \frac{f_\pi}{Q^2 + Q'^2} \,. \tag{3}$$

This is found for the $qq\gamma$ vertices which reduce to the bare γ^μ as soon as just one of the quark legs is hard, while the usage of the BC vertex again modifies this result by the substitution $f_\pi \to \widetilde{f}_\pi$. Eq. (3) agrees with the leading term of the OPE result derived by Novikov *et al.* [12] for the special case $Q^2 = Q'^2$. The distribution-amplitude-dependence of the pQCD approach cancels out for that symmetric case, so that $T_{\pi^0}(-Q^2, -Q'^2)$ in this approach (*e.g.*, see [13]), in the limit $Q^2 = {Q'}^2 \to \infty$, exactly agrees with both our Eq. (3) and Ref. [12]. Therefore, for that symmetric case, we should have even the precise agreement of the coefficients irrespective of the description of the pion internal structure encoded in the distribution amplitude. Obviously, this favors the $qq\gamma$ vertices which reduce to the bare γ^μ as soon as one of the quark legs is hard, over the BC vertex, and $\mathcal{K} = 4f_\pi/3$ over $\mathcal{K} = 4\widetilde{f}_\pi/3$. However, the BC vertex, which is the simplest WTI-preserving vertex and has been the one most widely used in phenomenological applications, may anyway be the one which is more accurate not only for the presently accessible Q^2, but also for much larger values before starting to fail. For that reason it is important to understand the asymptotic behavior to which the BC vertex leads.

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